Institute of Actuaries of India

ACET December 2018

Mathematics

1. The Characteristic roots of the matrix $\begin{pmatrix} 1 & 5 & 7 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$ are

   A. 0, 2, 4.
   B. 1, 2, 3.
   C. 1, 0, 2.
   D. 1, 5, 7.

   1 mark

2. If the matrix $\begin{pmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{pmatrix}$ is singular and x is real, then x must be equal to

   A. 1.
   B. -1.
   C. 0.
   D. either 1 or -1.

   2 marks

3. If $\log_9 (x+1) = \log_3 (x-1)$ and $x > 1$, then x is equal to

   A. e.
   B. 2.
   C. 3.
   D. 4.

   1 mark

4. If $\frac{2x+3}{(x+2)^2(x+1)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$, then A is equal to

   A. -1.
   B. 1.
   C. 5.
   D. 2.

   1 mark

5. The value of $\sqrt{2\sqrt{2\sqrt{2\ldots}}}$

   A. is 2.
   B. is greater than 2.
   C. is 1.
   D. cannot be found. 1 mark

6. The value of $\sin 105^{\circ}$ is

   A. $\frac{\sqrt{3}}{2\sqrt{2}}$.
   B. $\frac{1-\sqrt{3}}{2\sqrt{2}}$.
   C. $\frac{\sqrt{3}+1}{2\sqrt{2}}$.
   D. $\frac{\sqrt{3}}{\sqrt{2}}$. 1 mark

7. The sum of the finite series $1.2.3 + 2.3.4 + 3.4.5 + \cdots + n(n+1)(n+2)$ is

   A. $n^2(n+1)(2n+1)/6$.
   B. $n(n+1)(n+2)(n+3)/4$.
   C. $n(n+1)(n-2)(n-3)/6$.
   D. $n^4 + 2n^3 + 3n^2 + 2n$. 2 marks

8. The roots of the quadratic equation $2x^2 + x + 1 = 0$ are

   A. real valued and unequal.
   B. complex valued and unequal.
   C. real valued and equal.
   D. complex valued and equal. 1 mark

9. The values of x for the inequality $\frac{x^2-2x-1}{x+1} < x$ to hold are

   A. $(x > -1/3) \cup (x < -1)$.
   B. $(x > 1/3) \cup (x < -1)$.
   C. $(x > -1/3) \cap (x < -1)$.
   D. $(x < -1/3) \cap (x > -1)$. 3 marks

10. The unit vector in the direction of $3\vec{\imath} + 4\vec{\jmath} - 12\vec{k}$ is

A. $\left(-3\vec{\imath} - 4\vec{\jmath} + 12\vec{k}\right)/13.$

B. $\left(4\vec{\imath} + 3\vec{\jmath} + 2\vec{k}\right)/29.$

C. $\left(3\vec{\imath} + 4\vec{\jmath} - 12\vec{k}\right)/169.$

D. $\left(3\vec{\imath} + 4\vec{\jmath} - 12\vec{k}\right)/13.$ 1 mark

11. The positive integer *n* such that $\lim_{x\to 3} \frac{x^n - 3^n}{x-3} = 108$ is

A. 4.

B. 3.

C. 2.

D. 1. 2 marks

12. If $u = \log\left(\frac{x^2+y^2}{xy}\right)$ then $x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y}$ is

A. $\frac{x^2-y^2}{x(x^2+y^2)}$.

B. $\frac{y^2-x^2}{y(x^2+y^2)}$.

C. $\frac{x^2-y^2}{x(x^2+y^2)} + \frac{y^2-x^2}{y(x^2+y^2)}$.

D. 0. 2 marks

13. The value of the integral $\int_{-\pi/2}^{\pi/2} \left(\frac{\sin x}{2+\cos x}\right) dx$ is

A. 0.

B. $\log 2$.

C. $\log 4$.

D. 2. 1 mark

14. The value of the integral $\int \frac{1}{x+\sqrt{x}}\, dx$ is

A. $2\log(1+x) + c$.

B. $2\log\left(1+\sqrt{x}\right) + c$.

C. $\sqrt{2}\log(1+x) + c$.

D. $\sqrt{2}\log\left(1+\sqrt{x}\right)+c.$ 1 mark

15. The value of the integral $\int_{-1}^{1}\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots\right)dx$ is

A. $e-1.$
B. $e-e^{-1}.$
C. $e^2-1.$
D. $e^{-1}-e.$ 1 mark

16. The value of the integral $\int_0^{\pi/2}\sin^7 x\ dx$ is

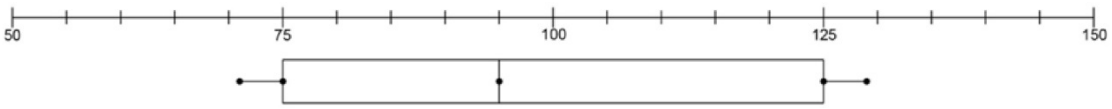
A. $2/3.$
B. $8/15.$
C. $24/35.$
D. $16/35.$ 3 marks

17. If $f(0)=-3$, $f(1)=6$ and $f(3)=12$, then the value of $f(6)$ by second degree polynomial fitting is

A. $114.$
B. $9.$
C. $-9.$
D. $11.$ 1 mark

Statistics

18. The following box plot has been drawn for the weights (in *lbs*) of a group of students in a class.



Then, the inter-quartile range and the median are, respectively,

A. 20 *lbs*, 95 *lbs*.
B. 20 *lbs*, 100 *lbs*.
C. 50 *lbs*, 95 *lbs*.
D. 50 *lbs*, 100 *lbs*. 1 mark

19. A discrete random variable takes positive integer values. If the probability of the value n is $ce^{-3}3^n/n!$, the value of c is

A. $1/(e^3-1)$.
B. $1/(1-e^{-3})$.
C. $1-e^{-3}$.
D. e^3-1. 1 mark

20. The arithmetic mean of the numbers $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$, where n is a positive integer, is

A. $2^n/n$.
B. $2^{n+1}/n$.
C. $2^{n+1}/(n+1)$.
D. $2^n/(n+1)$. 1 mark

21. A committee consisting of 5 persons is to be selected without replacement from a group of 6 men and 12 women. The probability that the selected committee would consist of 2 men and 3 women is

A. $\frac{1}{\binom{18}{5}}$.
B. $\frac{\binom{5}{2}}{\binom{6}{2}\binom{12}{3}}$.
C. $\frac{\binom{5}{3}}{\binom{18}{5}}$.
D. $\frac{\binom{6}{2}\binom{12}{3}}{\binom{18}{5}}$. 1 mark

22. An undergraduate course in Statistics offers two electives, A, B, to the students consisting of 30% females and 70% males. They have to opt for one of the electives. It is known that 80% male students register for elective A and rest for B and 60% female students register for elective A and the rest for B. The ratio of students who register for electives A and B is

A. $74 : 26$.
B. $80 : 60$.
C. $70 : 30$.
D. $66 : 34$. 2 marks

23. A continuous random variable X has pdf

$$f(x) = \begin{cases} c & \text{if } -2 < x < 2, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant. The variance of X is

A. $16/3$.
B. $4/3$.
C. $4c/3$.
D. $c/3$. 2 marks

24. Let A, B and C be events such that $P(A) = P(B) = 1/4$, $P(C) = 1/3$, $P(A \cap B) = 1/8$, $P(A \cap C) = 1/6$ and $P(B \cap C) = 0$. Then $P(A \cup B \cup C)$

A. is equal to $23/24$.
B. is equal to $17/24$.
C. is equal to $13/24$.
D. cannot be computed from the given information. 2 marks

25. Let A and B be subsets of the sample space S. Consider the following statements.
i. A is independent of S.
ii. If A and B are independent then A^c and B^c are also independent.
iii. If A and B are independent then A and B are mutually exclusive.
iv. If A and A^c are independent, then $P(A) = 0$ or 1.

Of these four statements,

A. i, ii and iii are correct.
B. i, ii and iv are correct.
C. ii, iii and iv are correct.
D. all are correct. 2 marks

26. Let X be a random variable with $E(X) = 10$ and $V(X) = 25$. Define $Y = aX - b, (a > 0$ and b are constants) such that $E(Y) = 0$ and $V(Y) = 1$. Then the pair (a, b) is equal to

A. $\left(1/\sqrt{10},\ 25/\sqrt{10}\right)$.

B. $(1/5\,, 2)$.

C. $(1/5\,,\ 1/2)$.

D. $(5, 50)$. 1 mark

27. A random variable X has *pdf* $f(x) = \frac{1}{2}\, e^{-|x|}$, $-\infty < x < \infty$. The *cdf* $F(x)$ of X is

A. $F(x) = \begin{cases} \frac{e^{-x}}{2} & \text{if } x < 0, \\ \frac{1}{2} + \frac{1}{2}\,(1 - e^{-x}) & \text{if } x \geq 0. \end{cases}$

B. $F(x) = \begin{cases} \frac{e^{x}}{2} & \text{if } x < 0, \\ \frac{1}{2} + \frac{1}{2}\,(1 - e^{x}) & \text{if } x \geq 0. \end{cases}$

C. $F(x) = \begin{cases} \frac{e^{x}}{2} & \text{if } x < 0, \\ \frac{1}{2} + \frac{1}{2}\,(1 - e^{-x}) & \text{if } x \geq 0. \end{cases}$

D. $F(x) = \begin{cases} \frac{e^{-x}}{2} & \text{if } x < 0, \\ \frac{1}{2}\,(1 - e^{-x}) & \text{if } x \geq 0. \end{cases}$ 3 marks

28. Suppose X is a normal random variable with mean μ and variance σ^2. Then $m\left[E\left(\frac{X-\mu}{\sigma}\right)\right]^2 + n\,V\left(\frac{X-\mu}{\sigma}\right)$ is equal to

A. $m + n$.
B. $m^2 + n$.
C. m.
D. n. 1 mark

29. The mean and the variance of a binomial random variable are 8 and 6, respectively. Then the number of independent trials is

A. 24.
B. 16.
C. 32.
D. none of these. 1 mark

30. Let X be an exponential random variable with mean 2. The coefficient of variation of X is

A. 1.
B. 2.
C. $1/2$.
D. 4. 1 mark

31. In a linear correlation study of two random variables X and Y, the following values are obtained.

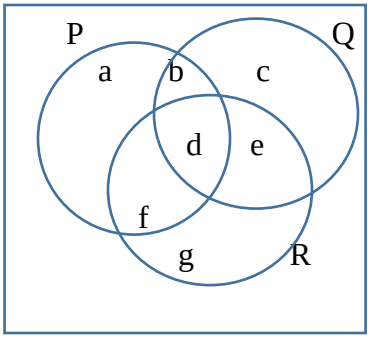
$$\bar{x} = 65, \quad \bar{y} = 67, \quad s_x = 2.5, \quad s_y = 3.5 \text{ and } r = 0.8.$$

Then the regression coefficient b_{yx} is computed as

A. 1.12.
B. 0.571.
C. -1.12.
D. -5.8. 1 mark

Data Interpretation

32. The elements of the sets P, Q and R are given in the Venn-diagram.



Then the set $(P \cap R) \cup (P \cap Q) \cup (Q \cap R)$ is given by

A. $\{a, b, d, e, f\}$.
B. $\{b, d, f, g\}$.
C. $\{a, d, c, e, f\}$.
D. none of these. 2 marks

33. A computer program implements the following algorithm.

Step 1 Let $A = 0$
Step 2 Let $B = A + 1$
Step 3 Let $C = A + 2B$
Step 4 Let $D = C^2 - (A + B)^2$
Step 5 Let $A = A + 1$
Step 6 Go to Step 2

After a few operations, the value of D was found out to be 304. What would be the value of A at this point?

A. 6.
B. 7.
C. 8.
D. Cannot be determined. 2 marks

Questions 34 to 36 are based on the information given in the table below about certain steel brands in India.

Name of the brand	Production ('000 tonnes)	Capacity Utilisation (%)	Sales ('000 tonnes)	Total Sales Value (Rs. In Crores)
AA	3.5	77	3.2	35
BB	2.8	73	2.3	28
CC	1.6	66	1.2	17
DD	1.5	61	1.1	19
Total (including others)	12.5	63	10	138

34. Which of the four brands mentioned above has the maximum production capacity?

A. AA.
B. BB.
C. CC.
D. DD. 2 marks

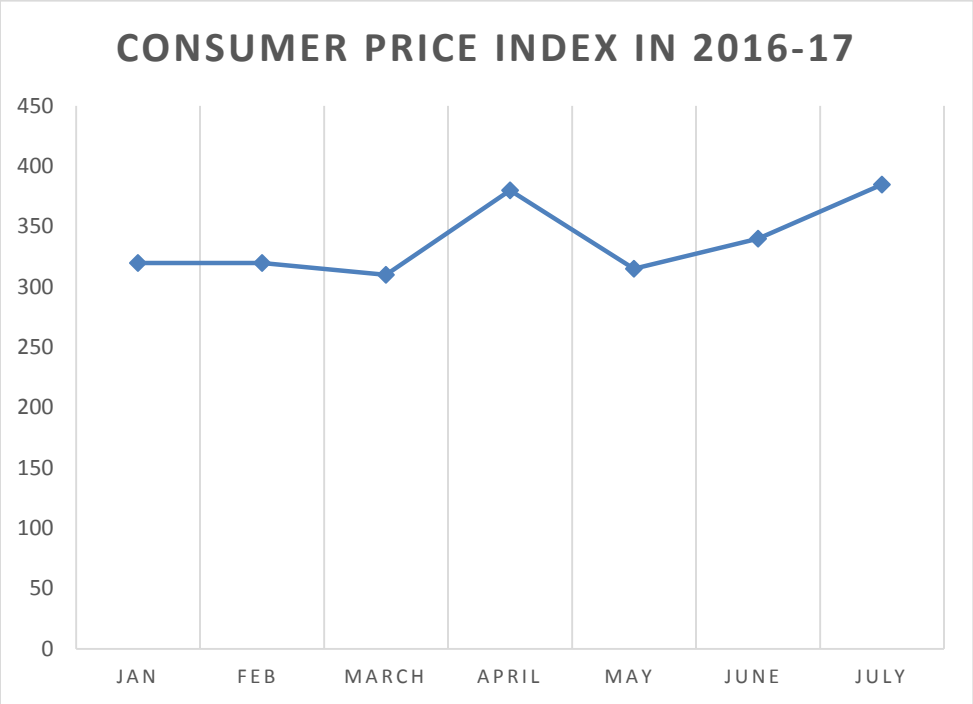
35. What is the unutilized production capacity (in tonnes) for the steel brand CC?

A. 1142.
B. 824.
C. 762.
D. 864. 1 mark

36. What is the average selling price of steel in India?

A. Rs. 13.8 lakh per tonne.
B. Rs. 1.38 lakh per tonne.
C. Rs. 0.72 lakh per tonne.
D. Data insufficient. 1 mark

Questions 37 to 38 are based on the information given in the chart below about consumer price index in India in the year 2016-17.



37. Which month showed the highest absolute difference in the Consumer Price Index (CPI) over the previous month?

A. July.
B. May.
C. April.
D. March. 1 mark

38. The difference in the number of months in which there was an increase in the CPI and the number of months in which there was a decrease was

A. Four.
B. Three.
C. Two.
D. One. 1 mark

English

39. ‘Misologist’ is a person who

A. hates to be in discipline.
B. hates to be known by many.
C. hates reasoned argument.
D. loves to be logical always. 1 mark

40. A ‘legend’ is not a story of

A. a supernatural being or event.
B. a mythological hero or god.
C. a very famous person.
D. a war between two kingdoms. 1 mark

41. A person who believes in fate is called

A. a fatalist.
B. an astrologer.
C. a fatal person.
D. a fatso. 1 mark

42. A person who walks in their sleep is called

A. an effeminate.
B. a kleptomaniac.
C. an insomniac.
D. a somnambulist. 1 mark

43. Select the word that cannot be used as a synonym of ‘Always’:

A. Invariably.
B. Forever.
C. Frequently.
D. Perpetually. 1 mark

44. Select the word that cannot be used as a synonym of ‘Modern’:

A. Advanced
B. Gothic
C. Neoteric
D. Innovative 1 mark

45. Select the word that cannot be used as an antonym of ‘Mitigate’:

A. Aggravate.
B. Palliate.
C. Exacerbate
D. Worsen. 1 mark

46. Select the word that cannot be used as an antonym of ‘Ambiguous’:

A. Vindicated.
B. Precise.
C. Exculpated.
D. Equivocal. 1 mark

47. Choose the most appropriate word to fill in the blank: “_____ the new day comes new strength and new thoughts.”

A. With.
B. As.
C. Along.
D. Before. 1 mark

48. Choose the most appropriate word to fill in the blank: "If you're going _____ hell, keep going.”

A. for.
B. in.
C. through.
D. with. 1 mark

49. Choose the most appropriate word to fill in the blank: “Because of the frequent power cut, the factories are forced to _____ workers in this area.”

A. fire off.
B. take off.
C. put off.
D. lay off. 1 mark

50. Choose the most appropriate word to fill in the blank: “People in a family must never _____.”

A. fall off.
B. fall out.
C. fall away.
D. fall apart. 1 mark

51. Select the pair which has relationship most similar to "Piano : Music":

A. Cable : Electricity.
B. Factory : Worker.
C. Sky : Sun.
D. Book : Knowledge. 1 mark

52. Select the pair which has relationship most similar to "Sun : Light":

A. Factory : Worker.
B. Tree : Wood.
C. Flower : Smell.
D. Spectacles : Eyesight. 1 mark

53. Select the pair which has relationship most similar to "Rice : Hunger":

A. Light : Darkness.
B. Lamp : Darkness.
C. Day : Darkness.
D. Rain : Drought. 1 mark

54. Select the right meaning of underlined phrase in the sentence: “Sometimes a very good speaker may also <u>go off on a tangent</u>.”

A. Talk rubbish.
B. Forget in between.
C. Make the speech boring.
D. Get off the subject. 2 marks

55. Select the right meaning of underlined phrase in the sentence: “The rat <u>took to its heels</u> when it saw a cat.”

A. Charged forward.
B. Went into hiding.
C. Broke down.
D. Ran away. 2 marks

56. Select the right meaning of the sentence: “He’s not playing with a full deck.”

A. He does not know how to play the game.
B. He is not showing his full capability.
C. He is very clever.
D. He is dumb. 2 marks

57. Select the right meaning of the sentence: “You can catch more flies with honey than you can with vinegar.”

A. You can expedite a matter through bribing.
B. You can get what you want by trapping and blackmailing someone.
C. Money gets you everything.
D. You will get what you want by being nice. 2 marks

58. Choose the correct sentence:

A. The new professor lectures above the heads of the students.
B. The easiest thing is to ask the address from the postman.
C. He never had and ever will take such strong actions against the corrupt.
D. Because of his mastery in his field, his suggestions have wide accepted.

2 marks

59. Choose the correct sentence:

A. Had you been told me about your problem, I would have helped you.
B. The police don't know how did the thief made an escape.
C. You need not offer any explanation; your behavior should speak for itself.
D. The husband cooks, washes dishes, sweeps the floor and then relaxing.

2 marks

60. Select the most logical order of sentences from among the given choices to construct a coherent paragraph:

P: Mobile headsets are cheaper as all the processing is done on your phone.

Q: There are two categories of modern VR headsets: Mobile or tethered.

R: A VR headset and motion tracking allows you to look around a virtual space without being there actually.

S: Virtual Reality is an amazing technology to travel in a virtual space.

A. RPSQ..
B. PQRS.
C. QSPR.
D. SRQP. 2 marks

Read the passage below and answer Question No. 61:

'Child labor' is violation of human rights, though it is considered by many in India to be a necessary evil. It hampers a child's physical, mental and social development. Children are employed in hotels, homes, workshops, shops, construction sites and so on. They even work in hazardous and unhygienic conditions in manufacturing factories. Article 24 of the Constitution of India, 1950 says, "No child below the age of fourteen years shall be employed to work in any factory or mine or engaged in any other hazardous employment". Indian legislature has also enacted the Factories Act, 1948, The Children Act, 1960, The Child Labor (Prohibition and Regulation) Act, 1986 etc. for the protection of rights of children. Article 45 of the Constitution of India, 1950 casts duty on the State to provide free and compulsory education to the children. Article 25(2) of the Universal Declaration of Human Rights also states that motherhood and childhood are entitled to special care and assistance. There can be no argument against the notion that child labor should be restricted and if possible completely eliminated. It is a socio-economic problem, which requires careful analysis and practical solutions.

I. The article that states about the special care and assistance for the mother and the child is

   i. Article 25(2) of the Universal Declaration of Human Rights.
   ii. Article 24 of the Constitution of India, 1950.
   iii. Both (i) and (ii) above.

II. As per Article 24 of the Constitution of India, 1950, a child below the age of fourteen years

   i. cannot work as domestic servants.
   ii. cannot work in mines.
   iii. Both (i) and (ii) above.

III. As per the constitution of India,

   i. Children's education is free and compulsory.
   ii. Mothers' education is free and compulsory.
   iii. Both (i) and (ii) above.

61. The correct answers to I, II and III are:

   A. iii, iii, iii, respectively
   B. i, ii, i, respectively
   C. i, ii, iii, respectively
   D. i, iii, i, respectively 3 marks

Read the passage below and answer Question No. 62:

One of the biggest problems faced by the world today is global warming. Many experts believe that our production of carbon dioxide and other greenhouse gases is heating the atmosphere, and this is very dangerous for human life. One of the worst consequences of global warming is rising sea level. This could result in the flooding of low-lying coastal areas, such as some parts of Egypt, the Netherlands, and Bangladesh. Change in the weather pattern is another problem. Many areas of the world are experiencing increased hurricanes, floods and other natural disasters. A third problem associated with this phenomenon is the adverse effect on animals. Fish populations could be affected, while some insects which spread disease might become more common. There are several things we can do to deal with global warming. One answer is to reduce emitting carbon dioxide. We can do this by switching from oil, coal and gas to renewable energy. A second solution is to plant more trees. Trees absorb carbon dioxide and produce oxygen, which is not a greenhouse gas. A third idea is to use less energy and recycle more products. If we use less energy and are more environment-friendly, the earth's temperature may not rise too much. Making small changes now in the way we live means avoiding huge changes in the future. Scientists, governments and individuals must work together to overcome this serious threat.

I. Select the statement that follows from the given passage:

i. Oxygen is a greenhouse gas.
ii. Global warming results in rising level of pollution.
iii. Bangladesh is a low-lying country.

II. Select the statement that does not follow from the given passage:

i. Global warming results in increased level of greenhouse gases.
ii. Changes in the weather patterns is the effect of global warming.
iii. Decreasing carbon dioxide may help in dealing with global warming problem.

III. The given passage does not indicate that global warming can be controlled by

i. planting more trees.
ii. controlling water levels in rivers.
iii. shifting to renewable energy.

62. The correct answers to I, II and III are:

A. iii, i, ii, respectively.
B. i, i, iii, respectively.
C. ii, i, ii, respectively.
D. ii, ii, ii, respectively. 3 marks

Logical Reasoning

63. If X is the brother of the son of Y`s son, how is X related to Y?

A. Son
B. Father
C. Cousin
D. Grandson 1 mark

64. The second day of a month is Sunday. What will be the last day of the next month which has 31 days?

A. Thursday.
B. Friday.
C. Tuesday.
D. Cannot be determined from the given information. 1 mark

65. Arrange the words given below in a meaningful sequence.

1. Heel 2. Shoulder 3. Neck 4. Knee 5. Chest 6. Thigh 7. Face

A. 3, 6, 7, 2, 4, 5, 1.
B. 7, 3, 2, 5, 6, 4, 1.
C. 3, 7, 2, 6, 5, 4, 1.
D. 7, 3, 2, 5, 4, 6, 1. 1 mark

66. Find the angle between the hour hand and the minute hand of a clock when the time is 3:30.

A. 75 degrees.
B. 15 degrees.
C. 90 degrees.
D. 45 degrees. 1 mark

67. All the faces of a cube are painted blue. The cube is cut into 64 small cubes of equal dimensions. How many small cubes have three faces painted blue?

A. 4.
B. 8.
C. 16.
D. 24. 1 mark

68. A statement is followed by two conclusions numbered I and II. You have to assume everything in the statement to be true, then consider the two conclusions together and decide which of them logically follows.

Statement: Forty percent of the national income is shared by the top five per cent of households in India.

Conclusions:

I. When an economy grows fast, concentration of wealth in certain pockets of population takes place.

II. The national income is unevenly distributed in India.

A. Only conclusion I follows.
B. Only conclusion II follows.
C. Both I and II follow.
D. Neither I nor II follows. 1 mark

69. While searching for an Electrician, Desai met three locals – Amar, Akbar and Anthony – who always gave two replies to any question. Among them one is a truth teller, one is a liar and one is an alternator (those who alternatively tell the truth and lie or vice-versa). When Desai asked them, "Who among you is the Electrician?", their replies were as follows.

Amar: I am the Electrician. Akbar is a liar

Akbar: I am the Electrician. Anthony is a liar

Anthony: Akbar is the Electrician. Amar is a liar.

Who is the electrician?

A. Amar.
B. Akbar.
C. Anthony.
D. Cannot be determined from the given information. 2 marks

70. In a group of persons travelling in a bus, 6 persons can speak Punjabi, 15 can speak Urdu and 6 can speak Bengali. In that group, none can speak any language other than these three. If two persons in the group can speak exactly two languages and one person can speak all the three languages, then how many persons are there in the group?

A. 21.
B. 22.
C. 23.
D. 24. 2 marks