# Institute of Actuaries of India

Solutions for ACET MAY 2016

# Mathematics

1. D. $\frac{2\pi}{3}$.
2. A. $\left|\overrightarrow{AB}\right|$ =$\sqrt{(2-1)^2+(-4-2)^2+\{1-(-3)\}^2}=\sqrt{(1)^2+(-6)^2+(4)^2}=\sqrt{53}$.
3. C. Interpolation possible by solving $n+1$ linear equations for $n+1$ unknown coefficients.
4. C. For the function to be defined the value inside the root must be non-negative. That is, $x-1\geq 0$ implies the domain is $[1,\infty)$.
5. B. $f\circ g(x)=f(g(x)=f(x^2)=x^2$+1. When x=3. $f\circ g(3)=10$.
6. D. by definition.
7. D. $\frac{1}{(x-1)(x+1)}=\frac{A}{(x-1)}+\frac{B}{(x+1)}$ implies $1=A(x+1)+B(x-1)$

   Putting $x=-1$ we have $B=-\frac{1}{2}$ and by putting $x=1$ we have $A=\frac{1}{2}$. Hence, $\frac{1}{(x-1)(x+1)}=\frac{1}{2(x-1)}-\frac{1}{2(x+1)}$.
8. A. The roots of $ax^2+bx+c=0$ are given by $=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Identifying the coefficients and constant in $2x^2+7x-4=0$ we have $x=-4,\ \frac{1}{2}$.
9. B. $e^3$ by definition.
10. B. $y=x^{-\frac{1}{2}}+\log_5 x\ +6$. Then $\frac{dy}{dx}=\frac{1}{2}x^{-\frac{3}{2}+\frac{1}{x}}\log_5 e$,

    since $\log_5 x=\log_e x/\log_e 5=\log_e x\log_5 e$.
11. D. $f(x)=x^3-3x+1$ so $f'(x)=3x^2-3$. In order that $f(x)$ decreasing in $x$ in an interval $f'(x)<0$. Hence $3x^2-3<0$ implies $x^2-1<0$ which is valid if $x\epsilon(-1,1)$.
12. C. For $a>0$, $\int_{-a}^{a}f(x)\,dx$ =2 $\int_0^a f(x)\,dx$ follows by properties definite integrals.
13. D. Put $\cos x=t$. Then $-\sin x\,dx=dt$. Further , when $x=0$ , $\cos 0=1$ and when $x=\frac{\pi}{2}$, $t=0$. Hence,

$$\int_0^{\frac{\pi}{2}}\frac{\sin x}{1+\cos^2 x}\,dx=\int_0^1\frac{dt}{1+t^2}=[\tan^{-1}t]_0^1=\tan^{-1}1-\tan^{-1}0=\frac{\pi}{4}.$$

14. B. The eight square matrices $A$ of order 3 satisfying $A^2 + I = 0$ are $A = \begin{bmatrix} \pm i & 0 & 0 \\ 0 & \pm i & 0 \\ 0 & 0 & \pm i \end{bmatrix}$. The matrix given in option B is one of these eight choices (plus or minus sign can be used in any of the three places).

15. C. If $A$ = [2 0 1] then $AA^T$ = 5. Hence the rank is 1.

16. C. $\frac{9-4}{5-3} = \frac{y-4}{4-3}$ implies $y = 4 + \frac{5}{2} = 6.5$ mm.

17. B. $\left(1+\sqrt{5}\right)^3 = 1 + 3\sqrt{5} + 3\sqrt{5}^2 + \sqrt{5}^3$ and $\left(1+\sqrt{5}\right)^3 = 1 - 3\sqrt{5} + 3\sqrt{5}^2 - \sqrt{5}^3$.

By adding, we have $= 2\left(1 + 3\sqrt{5}^2\right) = 32$.

18. D. The repayments are in AP. $a, a+d, a+2d, \ldots$.

The $n^{\text{th}}$ instalment is $T_n = a + (n-1)d$.

Here, $a = 1000,\ d = 50$. Therefore, $T_{30} = 1000 + (30-1)50 = 2450$.

# Statistics

19. A. If $\binom{n}{6} = \binom{n}{4}$, then $n$ must be 10. So $\binom{12}{n} = \binom{12}{10} = \binom{12}{2} = 66$.

20. C. The shoe sizes of 20 students, after decoding the dot plot are (in ascending order)

5 6 6 6 6 6 6 7 7 7 7 8 8 8 8 8 9 9 9 10.

The mode is 6; the median (middle most number) is 7, and the mean is certainly larger than 7. This last fact the apparent from the dot plot, as the centroid of the data has to be to the right of 7.

21. C. The mean is zero. The standard deviation is the square root of the sum of squares of the items divided by the number of items.

22. D. Let $B$ be the event that the electric relay is defective. Let $A_i\ \ i = 1,2,3$ be the event that the electric relay is manufactured in plant *i*.

$$P(A_1) = 0.5,\ \ P(A_2) = 0.3, P(A_3) = 0.2$$

Hence, $P(B|A_1) = 0.02$, $P(B|A_2) = 0.05$, $P(B|A_3) = 0.01$

So, $P(B) = P(B|A_1)P(A_1) + P(B|A_2)\,P(A_2) + P(B|A_3)\,P(A_3)$

$= (0.02 \times 0.5) + (0.05 \times 0.3) + (.01 \times 0.2) = 0.027$.

23. C. $A \cup B = (A \cap B^c) \cup B$ and P$(A \cap B^c) \cap B = \varphi$. So, $P(A \cup B) = P(A \cap B^c) + P(B)$. Hence, $P(B) = 1/3$.

24. B. $\int_0^1 kx(1-x)dx = 1$ implies $k\int_0^1 x(1-x)dx = k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{k}{6} = 1$. So, $k = 6$.

25. B. The mean and variance of the exponential distribution are $1/\mu$ and $1/\mu^2$, respectively.

$$\text{Coefficient of Variation } = \frac{\text{standard deviation}}{\text{mean}} = \frac{1/\mu}{1/\mu} = 1.$$

26. C. $X$ is Poisson and $P(X = 0) = 2P(X = 1)$ implies $\mu = 1/2$. Hence variance is $1/2$.

27. D. Follows from the properties of the correlation coefficient.

28. A. $b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{2}{3} \times \frac{2}{12} = \frac{1}{9}$.

# Data Interpretation

29. D. September.

30. B. August and November.

31. B. Trade balance = Exports – Imports. The aggregate for the year is about –10 million.

32. B. $227/205 \approx 1.1$.

33. C. August (deficit is $227 - 218 = 9$).

34. B. $(5820/16\%) \times 67\% = 24370 \approx 24400$.

35. C. 11% + 2% = 13% ( 11% for NRI and 2% for National Banks).

36. C. $2\% \,/\, 34\,\% = 1{:}17$.

37. C. $360° \times 11\% = 39.6° \approx 40°$.

38. A. $0.32x \,/\, 1.16x = 27.6\%$.

# English

39. C.

40. B.

41. A.

42. B.

43. B.

44. C.

45. A.

46. C.

47. C.

48. D.

49. A.

50. C.

51. B.

52. D.

53. C.

54. C.

55. B.

56. C.

57. C.

58. D.

59. C.

60. A.

61. D.

62. A.

63. C.

# Logical Reasoning

64. A. The man in the photograph is son of Mitali's grandfather's son i.e., the son of Mitali's father. Hence, the man is Mitali's brother.

65. D. Explanation: First Letter as it is. Second and Third Letter interchanged. Fourth and Fifth letter interchanged. Sixth and Seventh letter interchanged. Eighth letter as it is.

66. A.

67. B. The watch gains 5 seconds in 3 minutes, which is 100 seconds in 1 hour (3600 seconds), or 1 minute in 36 minutes. From 7 AM to 4:15 PM on the same day, the clock has passed 9 hours and 15 min, i.e., 555 min ($37 \times 15$ min). The actual time that has passed must be $37 \times 15 = 540$ min. Thus, the correct time should be 4:00 PM.

68. B. This is a leap year. So, none of the next 3 years will be leap years. Each year will give one odd day so the day of the week will be 3 odd days beyond Monday i.e. it will be Thursday.

69. D.

70. C. We solve this problem by assuming that one of them is the robber and then check whose statement is true of false. We summarize the findings in a table.

| Robber → Statement ↓ | Salman | Aamir | Saif | Shahrukh |
|---|---|---|---|---|
| Salman | F | T | F | F |
| Aamir | F | F | F | T |
| Saif | T | T | F | T |
| Shahrukh | T | T | T | F |
| | | | Only column with a single T | |

Therefore, it must be Saif who did it.