Institute of Actuaries of India

Solutions for ACET OCT 2016

Mathematics

1. B. $f(x) = g(x)$ implies $3x^2 - 1 = x + 3$
   i.e. $3x^3 - x - 4 = 0$
   Thus , $x = \frac{4}{3}\ or\ -1$.

2. C. $f(x) = f(0) + \frac{x}{1!}\ f'(0) + \frac{x^2}{2!}\ f''(0) + \cdots$ implies

   $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$

3. A. Simple linear interpolation implies

   $\frac{40-30}{1.303-1.31} = \frac{35-30}{t-1.31}$. This gives $t = 1.3065$

4. B $\left(\sqrt{3}+1\right)^5 - \left(\sqrt{3}-1\right)^5$
   $= \left(\sqrt{3}\right)^5 + C_1^5\left(\sqrt{3}\right)^4 + C_2^5\left(\sqrt{3}\right)^3 + C_3^5\left(\sqrt{3}\right)^2 + C_4^5\left(\sqrt{3}\right)^1 + 1$
   $-[\left(\sqrt{3}\right)^5 - C_1^5\left(\sqrt{3}\right)^4 + C_2^5\left(\sqrt{3}\right)^3 - C_3^5\left(\sqrt{3}\right)^2 + C_4^5\left(\sqrt{3}\right)^1 - 1$

   = 2[(5×9)+ (10×3)+1]=152

5. D $\frac{3x+7}{x^2-3x+2} = \frac{A}{x-2} - \frac{B}{x-1}$ implies$3x + 7 = A(x-1) + B(x-2)$

   Putting $x = 1$ gives $B = -10$ and $x = 2$ gives $A = 13$

   Thus $\frac{3x+7}{x^2-3x+2} = \frac{13}{x-2} - \frac{10}{x-1}$.

6. C. $\lim_{x\to\infty} \frac{x^2}{e^x} = \frac{\infty}{\infty}$ (indeterminate form)
   Using $L'Hospital's$ rule $\lim_{x\to\infty} \frac{2x}{e^x} = \frac{\infty}{\infty}$ (indeterminate form)
   Again using $L'Hospital's$ rule $\lim_{x\to\infty} \frac{2}{e^x} = 0$.

7. B. $y = \tan^{-1} x$ implies $x = \tan y$.
Hence, $\frac{dx}{dy} = \sec^2 y = 1 + \tan^2 y = 1 + x^2$ and $\frac{dy}{dx} = \frac{1}{1+x^2}$.

8. A. $\frac{d}{dx} \log(\log x) = \frac{1}{x \log x} > 0 \; if \; x > 1$
Hence log (log *x*) is increasing in *x* in [0,∞)

9. A. $\int_0^1 \frac{e^x+1}{e^x} dx = \int_0^1 1 + e^{-x} \, dx = x - e^{-x}]_0^1 = 2 - e^{-1}$

10. B. Given : $x^3 + 8xy + y^3 = 64$.
Therefore $3x^2 + 8x\frac{dy}{dx} + 8y + 3y^2 \frac{dy}{dx} = 0$.
$\frac{dy}{dx}[8x + 3y^2] = -3x^2 - 8y$
Hence, $\frac{dy}{dx} = \frac{-3x^2-8y}{8x+3y^2}$.

11. B. $\int \log x \, dx = x \log x - \int x \frac{1}{x} \, dx + k = x \log x - x + k$

12. C. $\int_0^\infty x^6 \, e^{-\frac{x}{2}} dx = \int_0^\infty (2y)^6 e^{-y} \, 2y \, dy$ substituting $y = \frac{x}{2}$

$= 2^7 \int_0^\infty y^{8-1} e^{-y} dy = 2^7 \Gamma 8 = 2^7 7!$

13. D. $\vec{a}.\vec{b} = (2\vec{\imath} + 2j - \vec{k})(\, 6\vec{\imath} - 3j + 2\vec{k}) = [(2 \times 6) + (2 \times (-3)) + ((-1) \times 2)]$
$= 4$

14. A. $\vec{a} \parallel \vec{b}$ implies $\vec{a} = t\vec{b}$ for some scalar *t*.
So $3\vec{\imath} + 2j + 9\vec{k} = t(\vec{\imath} + mj + 3\vec{k})$
Equating the coefficients, we have, $mt = 2 \; and \; t = 3$. Thus $m = \frac{2}{3}$.

15. C. $P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Therefore $P^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$

and $P^3 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix} = 9P$. Likewise $P^4 = 27P$.
The answer follows.

16. A. Since $Adj \; P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $|P| = 1$, $P^{-1} = P^T$

17. D. $P=\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$, $P^T=\begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix}$ and $P=P^T$ implies

$22x-3=x+2$ giving x = 5.

18. D. Any two rows of order 2 and 3 are *identical*. Hence the rank is 1.

Statistics

19. D. C_r^{15} : C_{r-1}^{15} = 11: 5 impliesC_r^{15} / C_{r-1}^{15} = 11/ 5**.**

i.e $\frac{\frac{15!}{r!(15-r)!}}{\frac{15!}{(r-1)!(15-r+1)!}}=\frac{11}{5}$. This gives $\frac{16-r}{r}=\frac{11}{5}$. Hence r = 5.

20. A. The middle most number of the 15 observation is 8th observation. Hence median is 3.

21. C. Let x and y be the numbers. Then $\frac{x+y}{2}=25$ and $\sqrt{xy}=15$.

This implies $+y=50$ and $xy=225$. Substitution gives $y^2-50y+225=0$.

Solving this equation gives the numbers as 45 and 5.

22. A. $P(A\cup B)=P(A)+P(B)-PA\cap B)$ =$0.8+0.35-0.25=$ 0.9

23. C $P(X\geq 1)=1-P(X<1)=1-P(X=0)=1-e^{-\theta}=1-e^{-2}$

Hence the mean is 2.

24. B. Using the formula for sum to n terns of the natural numbers and sum of squares of n natural numbers we get the solution.

25. A. $P\left(\frac{A}{A\cup B}\right)=\frac{P(A\cap(A\cup B)}{P(A\cup B)}=\frac{P(A)}{P(A\cup B)}=\frac{P(A)}{P(A)+P(B)-P(A)P(B)}=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{3}{4}-\left(\frac{1}{3}\times\frac{3}{4}\right)}=$

= 2/5.

26. B. 2 Using simple integration or identifying as the density of Triangular distribution.

27. B. $P(X < 46) = P\left(\frac{X-40}{4} < \frac{46-40}{4}\right) = P(Z < 2.5) = 0.9332$. [Std normal tables not needed, as none of the other options do not even come close.]

28. D. It is easy to show that $\sigma_Z = 20$ and $\bar{Z} = 205$.
The regression equation Z on X is $(Z - \overline{Z}) = \frac{r\sigma_Z}{\sigma_X}(X - \bar{X})$.
Substituting the values we get $Z - 205 = 8\ (X - 10)$.

Data Interpretation

29. C Between 2014 and 2015. This can be visually seen as we have negative profit in 2015 while the profit in 2014 was higher than that in 2016.

30. B. Interest . This can be visually observed very easily.

31. B. Wages remained nearly constant from 2013 to 2016. Therefore, among these four years, wages(as a percentage of the total cost of production) was the largest in 2013.From 2012 to 2013, the wage rose faster than the total costs.

32. A. West Bengal.

33. B 0%.

34. C. 14 . The average production of Andhra Pradesh is 68 and the average production of Karnataka is 54. Therefore the difference is 14.

35. C. 486000 . (486000= 44% * 2700000* 1.25 - 37% * 2700000).

36. D. 3. The models with higher than average sales would have larger market share (in terms of number if vehicles sold) in 2016. Comparison of the pie charts show that the shares of Scorpio, Bolero and Mahindra XUV increased in 2016 while that of Fortuner and Tata sumo reduced.

37. A. 2016. The ratio comes to around 38.6 in 2016.

38. D. None

.

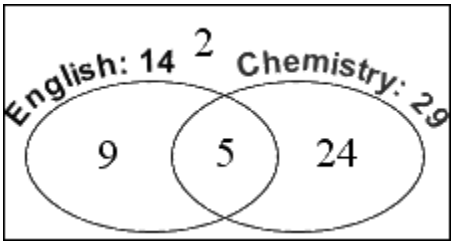
English

39. D
40. A
41. D
42. D
43. B
44. C
45. A
46. B
47. D
48. B
49. A
50. D
51. B
52. B
53. C
54. B
55. D
56. A
57. D
58. A
59. C
60. B
61. C
62. C

Logical reasoning

63. A. A group of fish is a school; a group of wolves is a pack.

64. C. Here, there are two alternating patterns. The first begins with 17 and adds 2; the second begins with 32 and subtracts 3.
65. A. The year 2008 was a leap year. So, it had 2 odd days. So, the day on 8th Dec, 2007 will be 2 days before the day on 8th Dec, 2008. But 8th Dec, 2008 is Monday. Hence, 8th Dec, 2007 is Saturday.
66. A. The letters of the word are written in a reverse order and then each letter is moved one step backward to obtain the code.
67. D.
68. A.



69. B. From Sunday noon to the following Sunday at 2 pm there are 7 days 2 hours or 170 hours. The watch gains $2 + 4\frac{4}{5} = \frac{34}{5}$ minutes in 170 hrs. Therefore, the watch gains 2 minutes in $2 \times \frac{5}{34} \times 170 = 50$ hours. Now, 50 hours means 2 days and 2 hours. 2 days and 2 hours from Sunday noon would be 2 pm on Tuesday.
70. B. Using i, the order is E---A. Then, by using iv, the order is E--HAB. Then, by using the information in ii, the order is E--HAB-D. Finally, by using the information in iii, the final order is EFGHABCD.